

Reflection Matrix Imaging in Wave Physics

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Institut Langevin, ESPCI Paris, CNRS, PSL University, Paris, France













Badon et al.,

Sci.

Adv.,

2016

2020

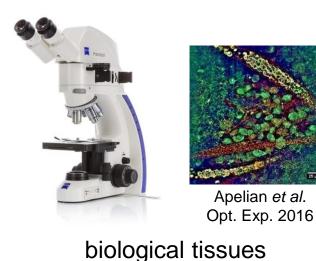


Blondel et al.,

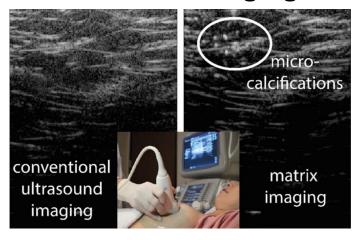
2018

Conventional wave imaging in inhomogeneous media

Optical microscopy



Ultrasound imaging



_ambert et al., PNAS, PRX, 2020

liver/breast imaging

Same limitations due to the medium inhomogeneities (aberrations/multiple scattering)

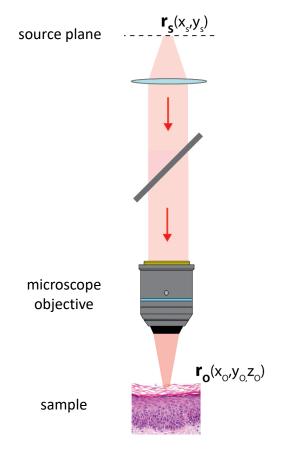
Our holy grail:

Universal and non-invasive approach of wave imaging in complex media

Introduction: Optical microscopy

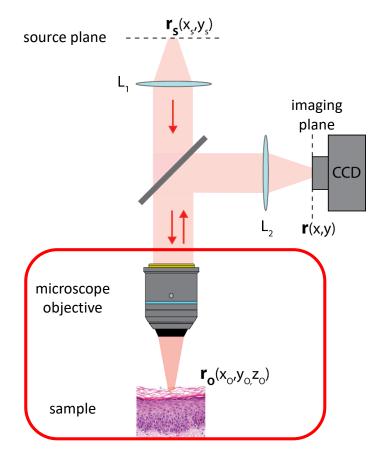


Principle of a microscope in reflection



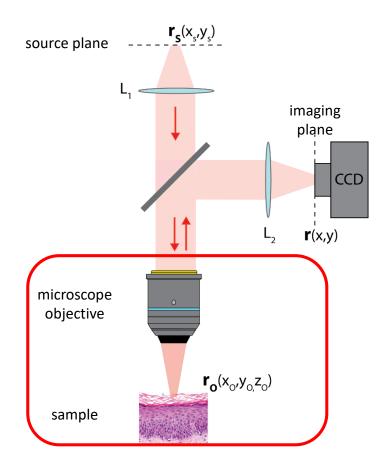


Principle of a microscope in reflection



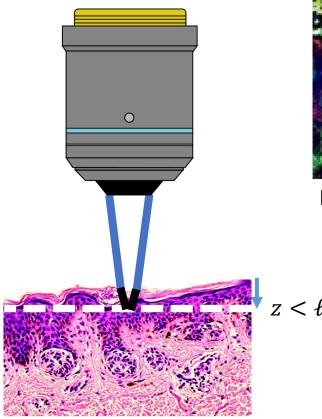


Principle of a microscope in reflection



SINGLE SCATTERING

BALLISTIC PATHS



Diffraction-limited resolution

 ℓ_s : scattering mean free path

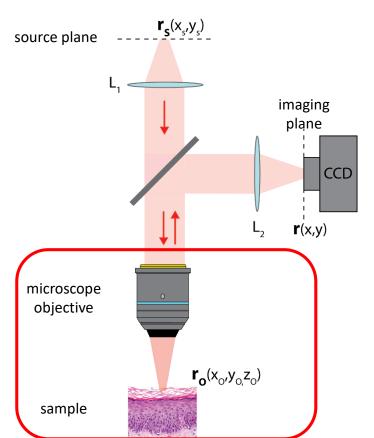
 $\ell_s \sim 50-200~\mu m$ in biological tissues

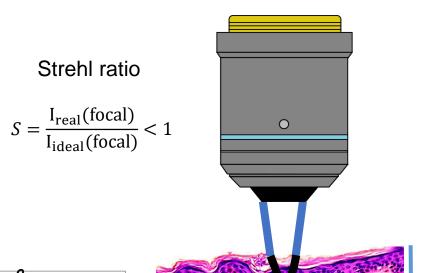


Principle of a microscope in reflection

ABERRATIONS / FORWARD MULTIPLE SCATTERING

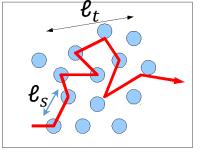
WAVE-FRONT DISTORTIONS

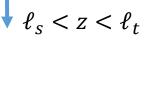






Loss of resolution and contrast

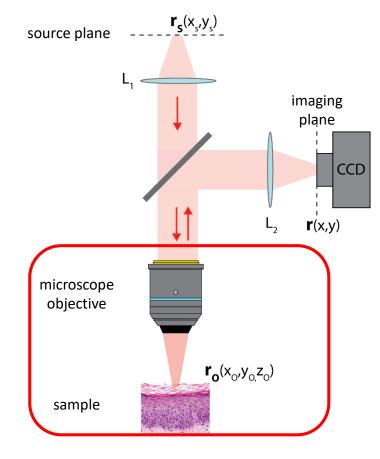


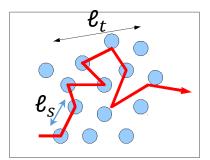


 ℓ_t : transport mean free path $\ell_t{\sim}1~\mathrm{mm}$ in biological tissues



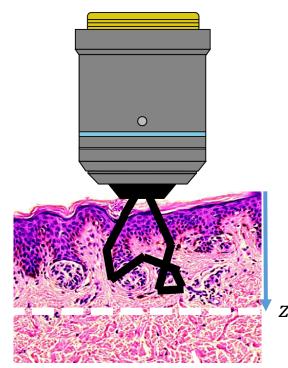
Principle of a microscope in reflection





DEEP MULTIPLE SCATTERING

RANDOM PATHS / DIFFUSION





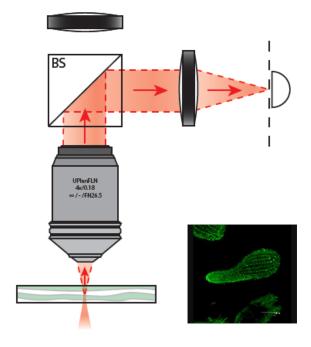
Random speckle image

 $z > \ell_t$



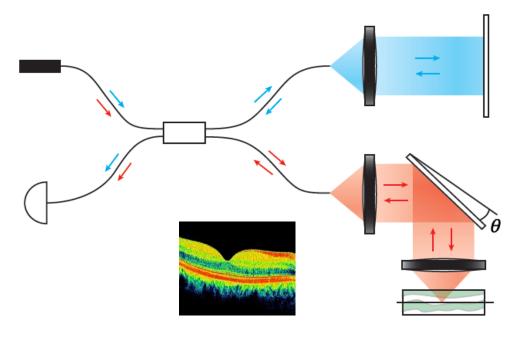
ENHANCE THE SINGLE SCATTERING CONTRIBUTION

Spatial discrimination
Confocal microscopy



M. Minski, Microscopy apparatus (1961)

Temporal discrimination
Optical coherence tomography

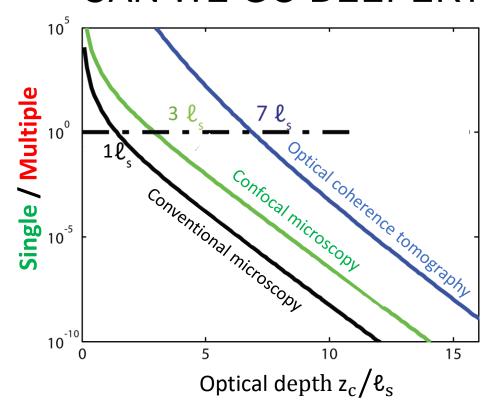


Huang, et al., science 254, 1178–1181 (1991)

Introduction: The multiple scattering limit in microscopy



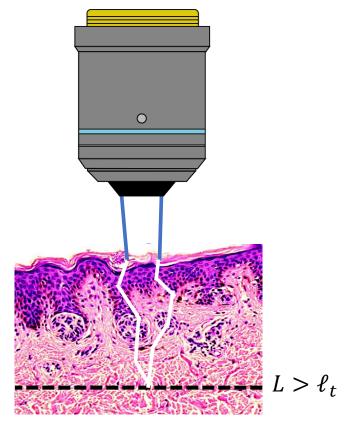
CAN WE GO DEEPER?



A. Badon et al., Opt. Exp., 2017

TACKLE WAVE DISTORTIONS

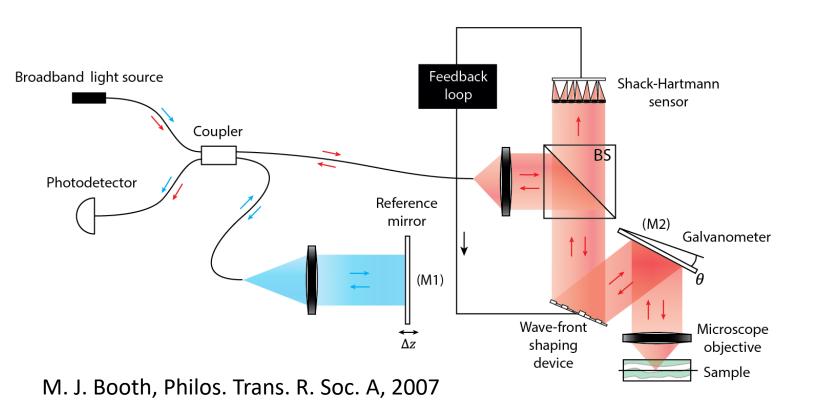
REMOVE MULTIPLE SCATTERING BACKGROUND



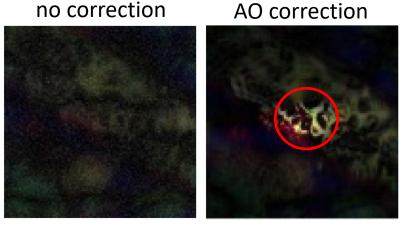
 ℓ_t : transport mean free path $\ell_t \sim 1 \text{ mm}$ in biological tissues

Introduction: Adaptive optics applied to optical microscopy





without aberrations

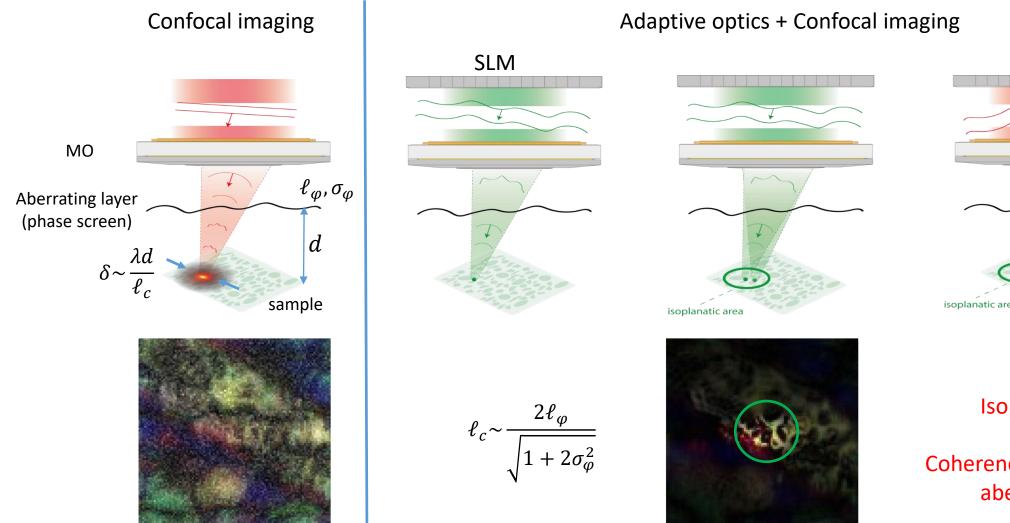


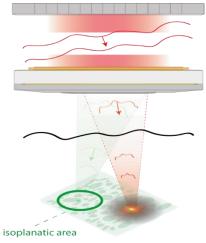
Isoplanatic area (few μ m at a depth of 1 I_t)

Adaptive focusing law only valid over a single isoplanatic patch

Introduction: Spatially-distributed aberrations







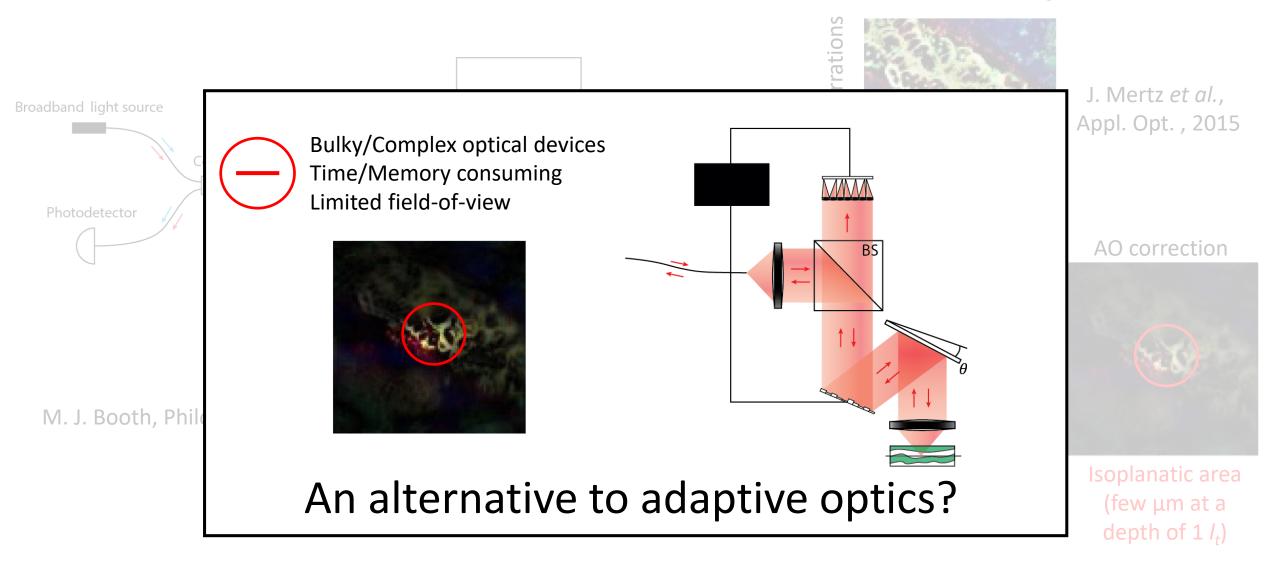
Isoplanatic area

Coherence length ℓ_c of the aberrating layer

Adaptive focusing law only valid over a single isoplanatic patch

Introduction: Adaptive optics applied to optical microscopy

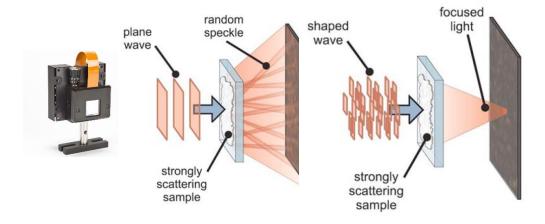




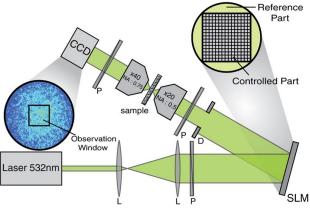
Introduction: Transmission matrix for optimal focusing



Focusing through strongly scattering media

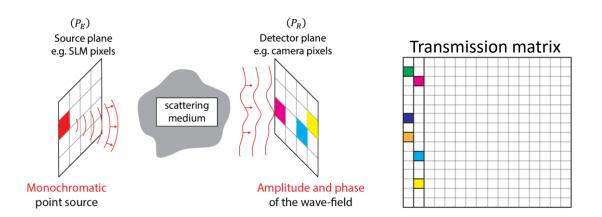


Vellekoop and Mosk, Opt. Lett. (2007)



Popoff et al., Phys. Rev. Lett. (2010)

Transmission matrixbetween the pixels of a SLM and of a CCD camera



Response matrix
$$\mathbf{T} = [T(\underbrace{SLM_i, CCD_j})]$$

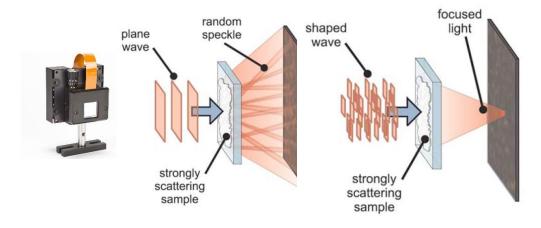
$$\begin{array}{c} \frac{\mathsf{S}}{\mathsf{S}} \\ \overset{\mathsf{S}}{\mathsf{S}} \\ \overset{\mathsf{S}}{\mathsf{S}} \\ & \\ t_{21} \\ \vdots \\ & \\ t_{N,1} \end{array} \qquad \begin{array}{c} \cdots & t_{1,N} \\ & \\ \vdots \\ & \\ t_{N,N} \end{array})$$

T contains all the information available on the medium

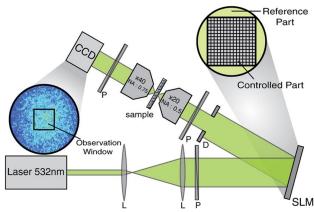
Introduction: Transmission matrix for optimal focusing



Focusing through strongly scattering media

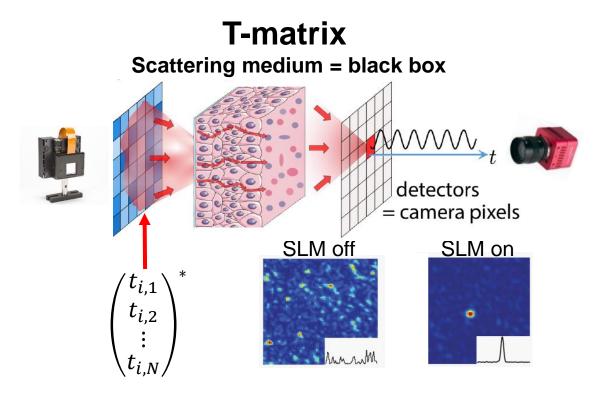


Vellekoop and Mosk, Opt. Lett. (2007)



Popoff et al., Phys. Rev. Lett. (2010)

Phase conjugation/inversion

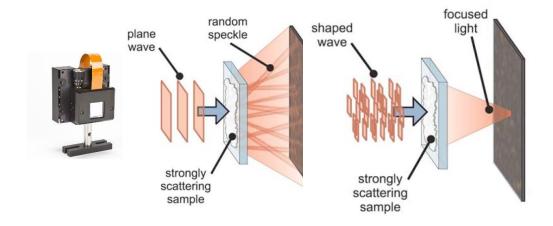


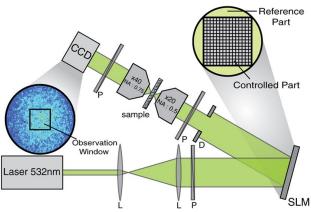
Any turbid medium can behave as a lens

Introduction: Transmission matrix for imaging?



Focusing through strongly scattering media

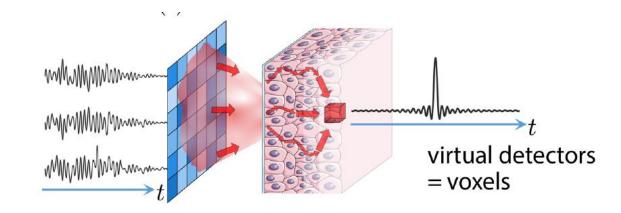




Popoff et al., Phys. Rev. Lett. (2010)

HOLY GRAIL FOR IMAGING:

TRANSMISSION MATRIX INSIDE THE MEDIUM



OPEN THE BLACK BOX BUT HIGHLY INVASIVE...

Introduction: Reflection matrix for imaging?

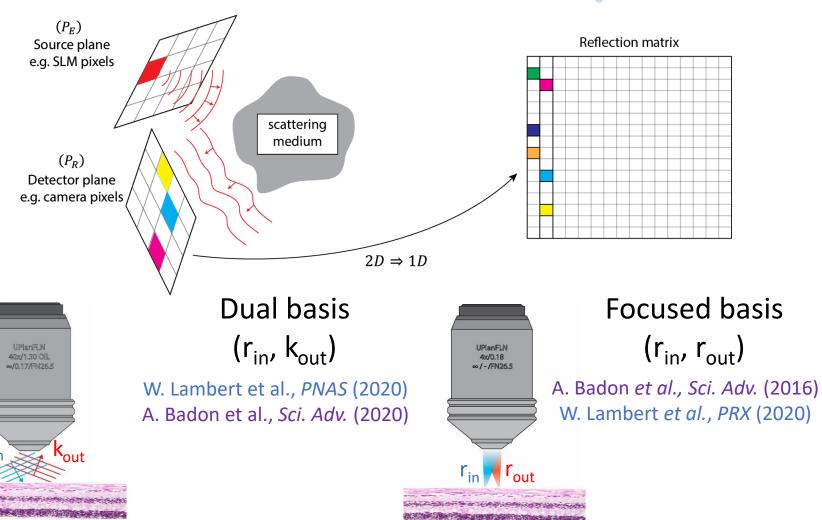


Reflection matrix:

From ultrasound imaging to optical microsocpy

« Far-field » basis (k_{in}, k_{out})

S. Kang *et al., Nat. Photon.,* 2015, 2017 M. Kim *et al., Nature Com.,* 2019



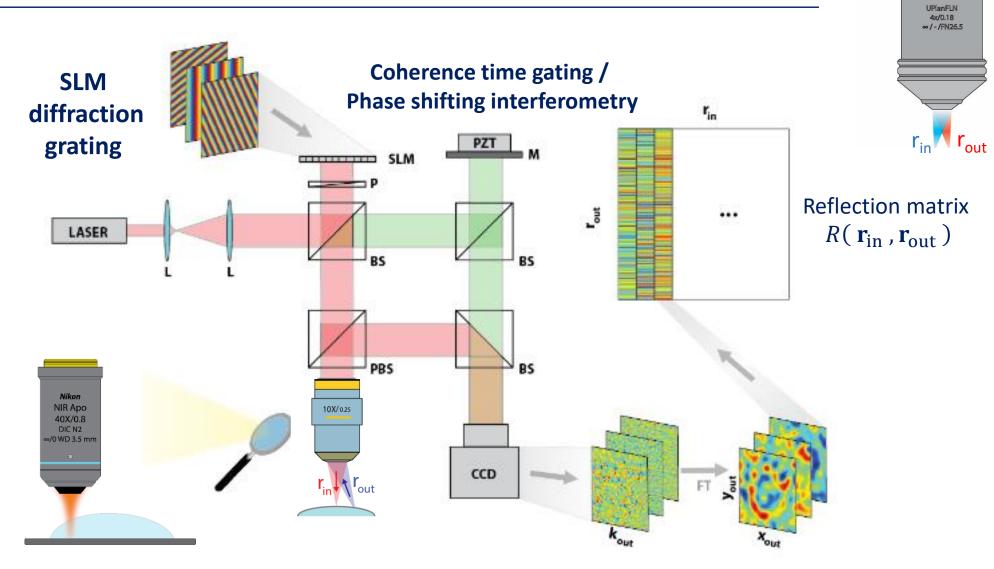
OBJECTIVE: Retrieve the TRANSMISSION matrix INSIDE the medium from a REFLECTION matrix recorded OUTSIDE?

Focused Reflection Matrix A coherent proof-of-concept

Coherent optical system: Time-gated reflection matrix

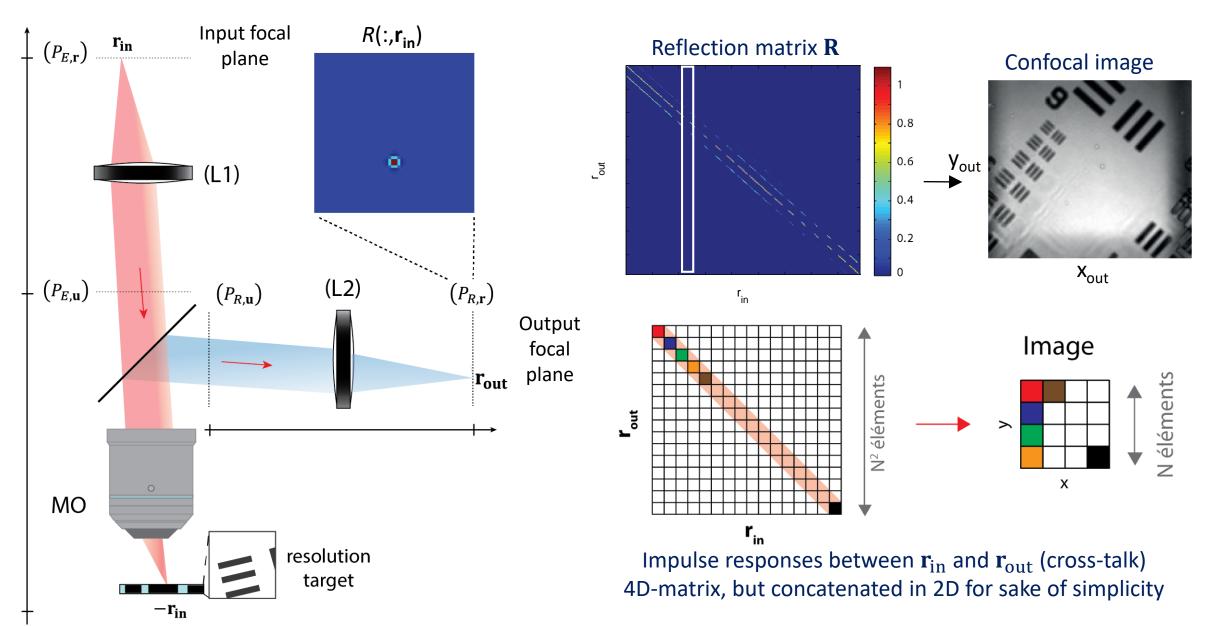


Amaury Badon



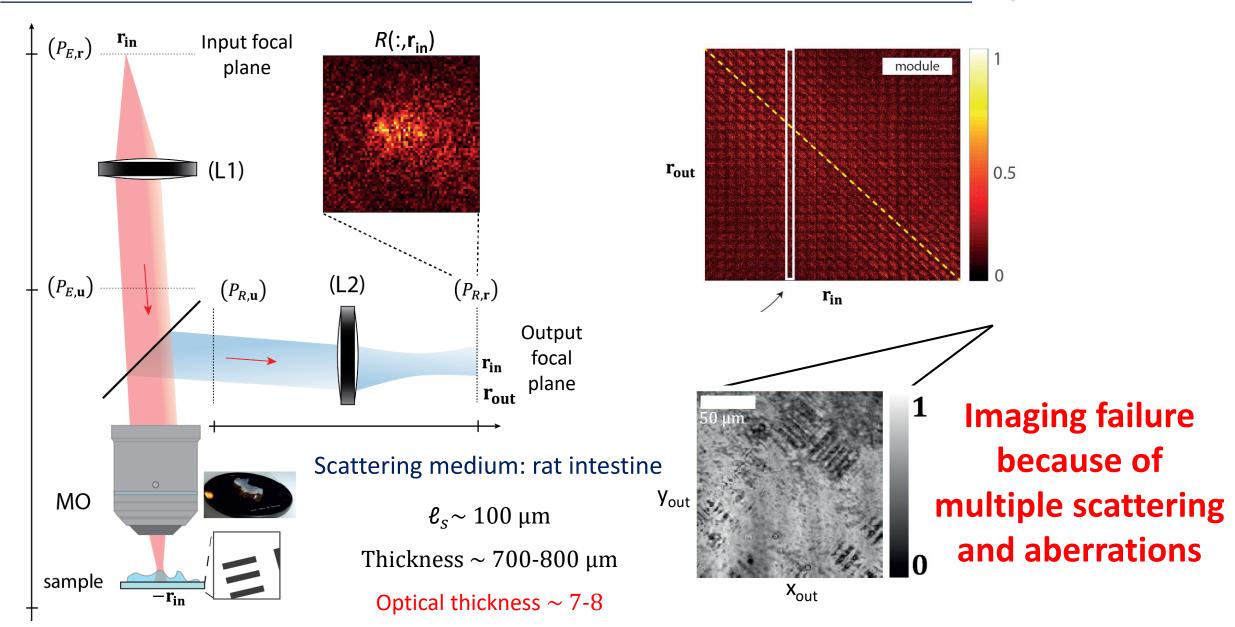
Focused reflection matrix





Focused reflection matrix

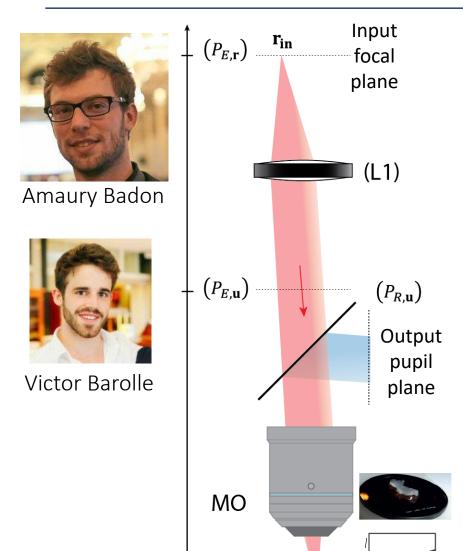




Distorsion Matrix A coherent proof-of-concept

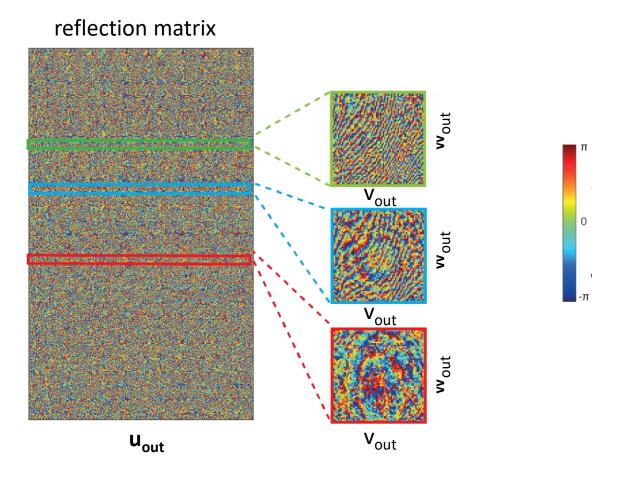
Dual reflection matrix





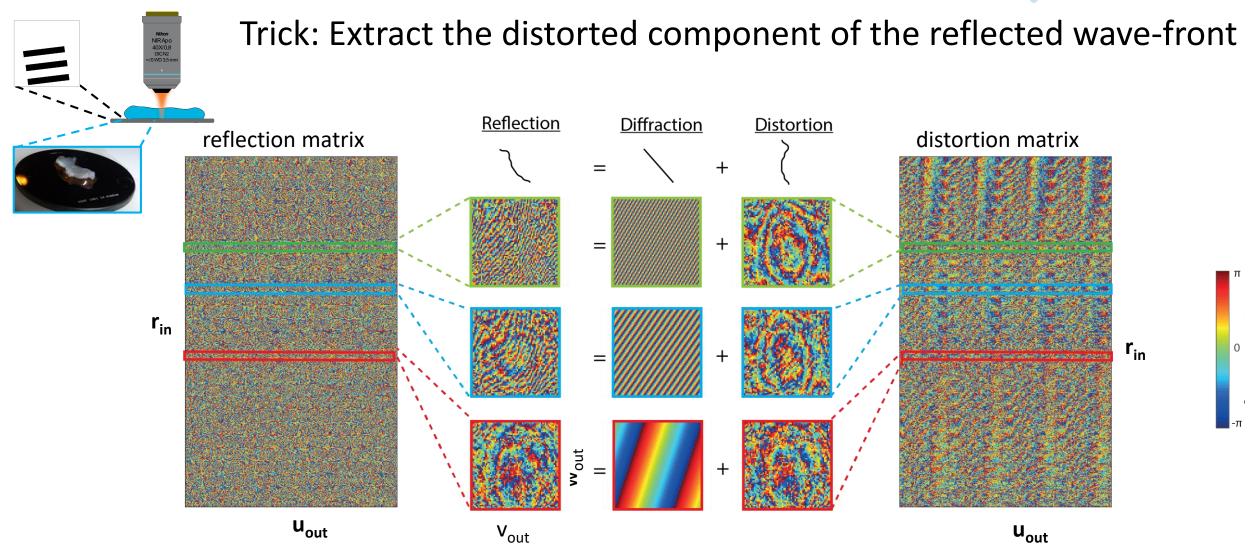
sample

Dual bases: Input focusing ←→ Output pupil



No correlations between the lines/columns of R





The distortion matrix reveals the hidden correlations of the reflected wave-field

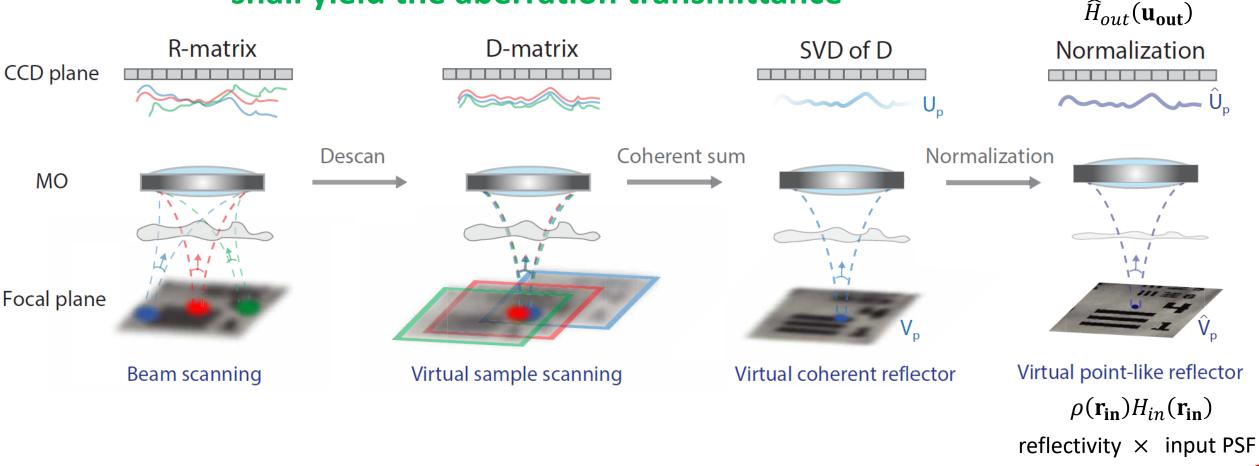
Covariance matrix / Time-reversal operator



pupil aberration

transmittance

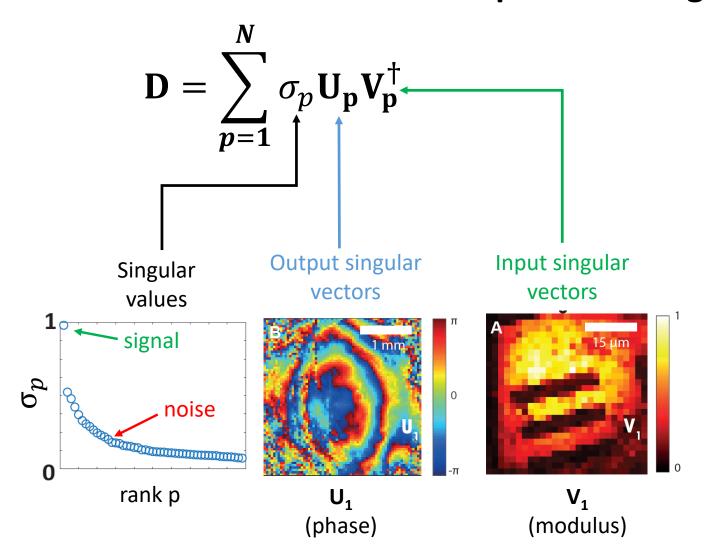
Correlations beween input focusing illuminations shall yield the aberration transmittance

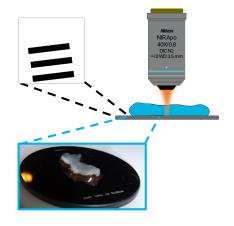


But need to average over a sufficiently large number of input focusing points : $N_{in} \gg (\delta_{in}/\delta_0)^2$



In practice, a singular value decomposition of **D** is performed to **remove diffuse multiple scattering** and **noise**





Isoplanatic case

Signal subspace of rank 1

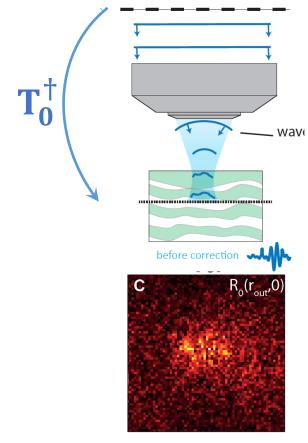
U₁: Pupil transmittance

 V_1 : Reflectivity \times input PSF

Conventional vs Matrix imaging



Conventional imaging

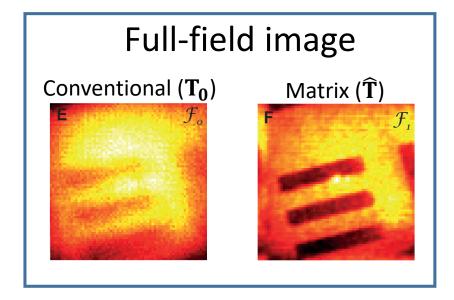


PSF width: $\delta{\sim}15~\mu{\rm m}$

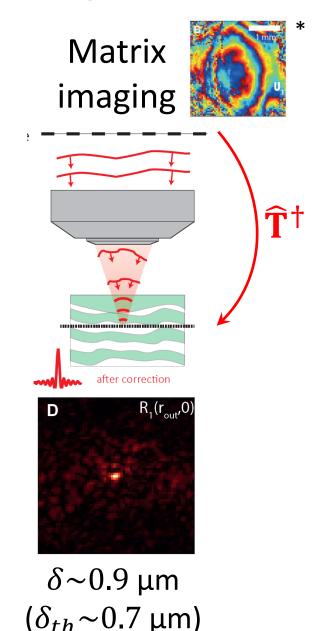
Free space T-matrix: T_0 (FT operator)

T-matrix estimator: $\widehat{\mathbf{T}} = \mathbf{T_0} \circ \mathbf{U_1}$

Focused R-matrix: $\mathbf{R}_{\mathbf{F}} = \widehat{\mathbf{T}}^{\dagger} \times \mathbf{R}$



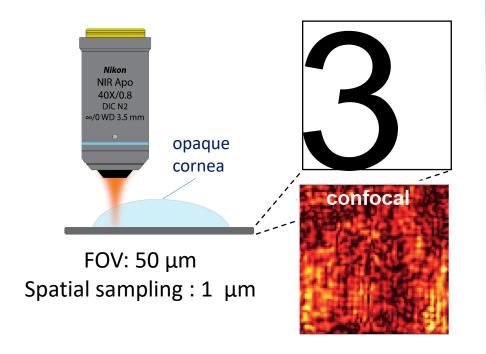
Strehl ratio increased by a factor 150 Resolution improved by a factor 15



Imaging through an opaque cornea: Beyond isoplanicity

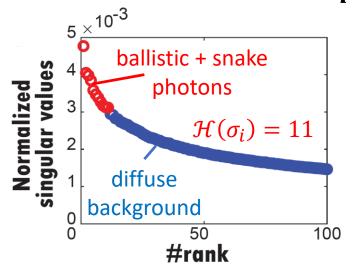


700- μ m-thick edematous primate cornea ($\ell_s \sim 70 \ \mu$ m) Optical thickness $\sim 10 \ell_s$



Singular value decomposition of **D**

$$\mathbf{D} = \sum_{p} \sigma_{p} \mathbf{U}_{\mathbf{p}} \mathbf{V}_{\mathbf{p}}^{\dagger}$$



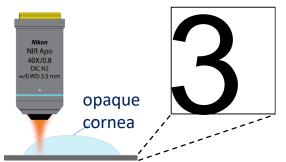
Effective rank of the imaging process = Entropy of singular values

$$\mathcal{H}(\sigma_i) = -\sum_{i=1}^N \sigma_i^2 \log_2(\sigma_i^2)$$

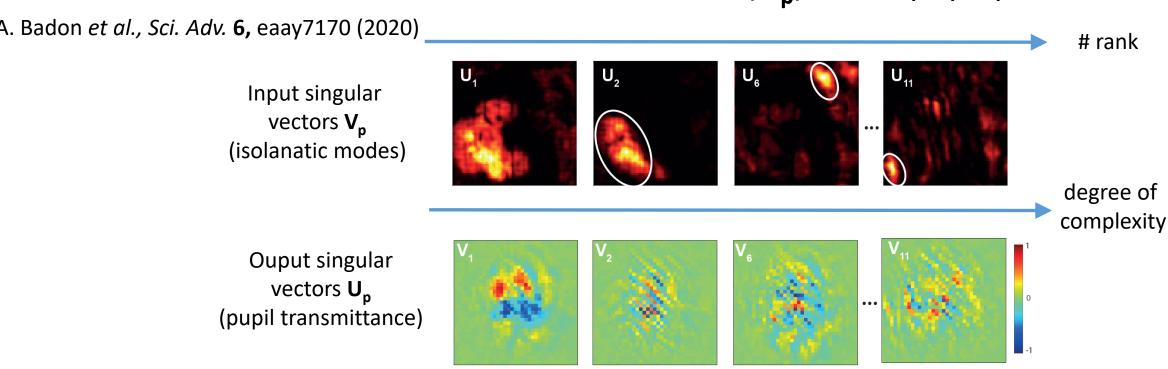
Entropy of D scales as the number of isoplanatic patches

Imaging through an opaque cornea





The SVD of **D** decomposes the field-of-view onto a set of orthogonal isoplanatic modes $(\mathbf{V_p})$, each one being associated with a different transmittance $(\mathbf{U_p})$ in the pupil plane

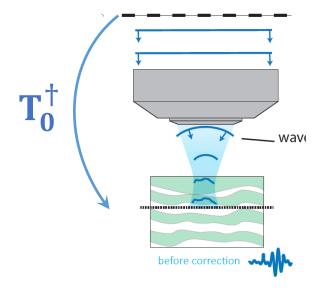


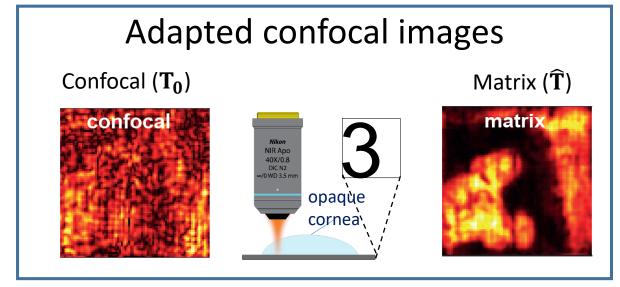
Adequate basis to characterize and quantify high-order aberrations



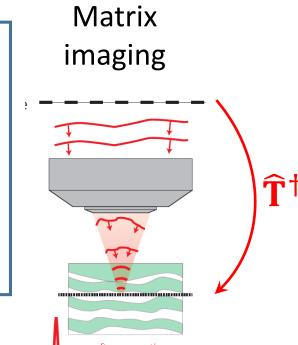
Coherent combination of pupil singular vectors: $\widehat{\mathbf{T}} = \mathbf{T_0} \circ \sum_{p=1}^{\mathcal{H}(\sigma_i)} \mathbf{U_p^{\dagger}}$

Conventional imaging





Strehl ratio increased by a factor 230



Full-field aberration correction (beyond the isoplanatic hypothesis)



Deep matrix imaging:

Correlation-based discrimination between

- single scattering paths
- forward multiple scattering paths
- random walks

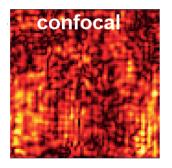
Computational approach (post-processing)

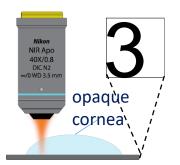
Overcome the intrinsic limit of adaptive optics (isoplanicity) Penetration depth > 10 I_s

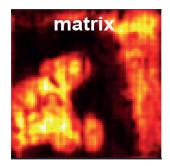
Normalized Single Nikon Nikon

Coherent optical system:

Academic proof-of-concept Invasive - Scanning technique Limited FOV ($\sim 10^3$ pixels) / Long acquisition time Moderate axial sectioning ($\sim 10~\mu m$) Only 2D imaging of specular reflectors so far...







Passive matrix imaging

An optical proof-of-concept

Passive matrix imaging: Full-field OCT

Incoherent optical system derived from full-field optical coherence tomography

Non-invasive
Full-field configuration
Huge FOV (10⁹ voxels) / Moderate acquisition time
Excellent lateral and axial resolution



Victor Barolle



Paul Balondrade

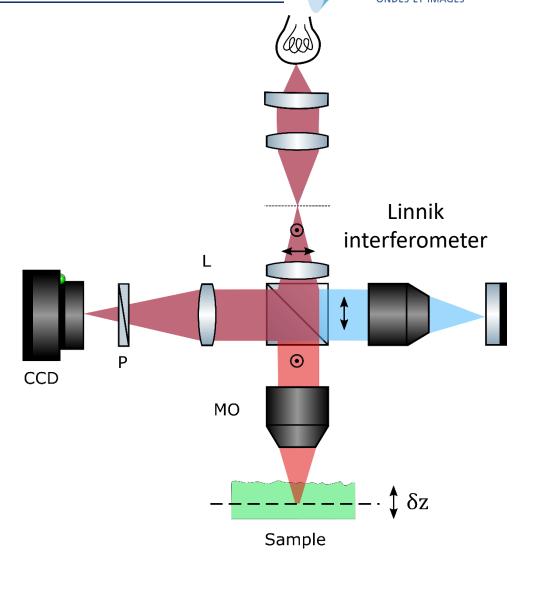


Ulysse Najar



A. C. Boccara



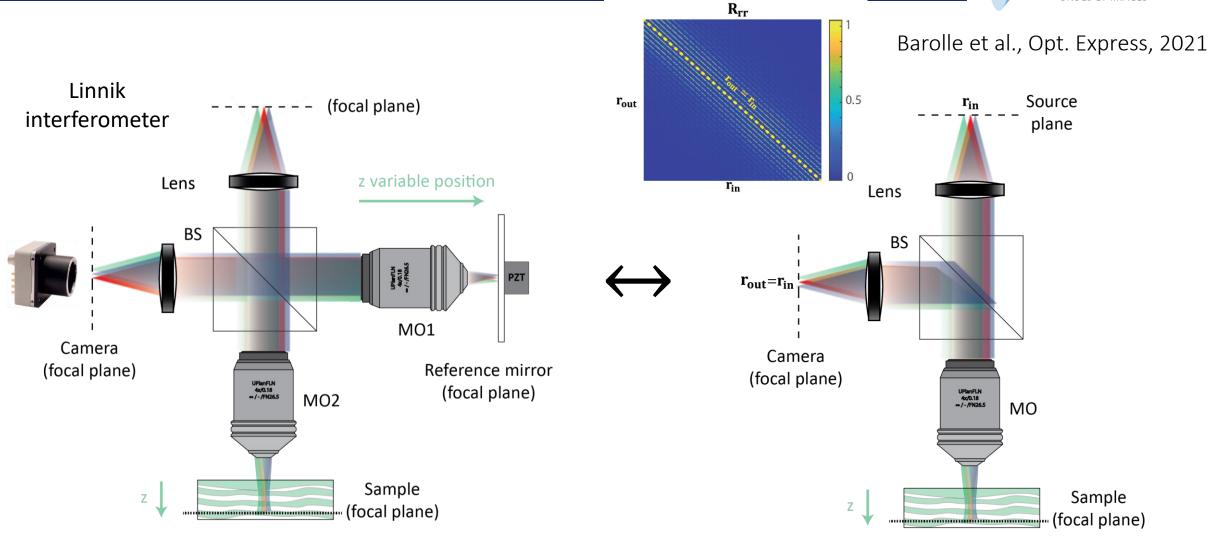


E. Beaurepaire, A. C. Boccara, M. Lebec, L. Blanchot, and H. Saint-Jalmes, Opt. Lett., 1998

Institut **Langevin**

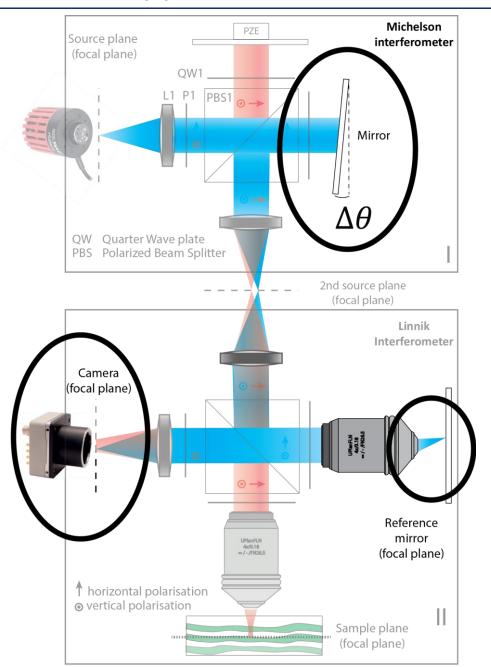
Full-field optical coherence tomography

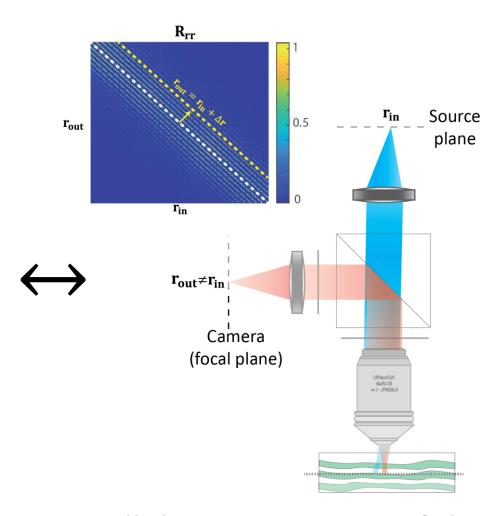




Parralel measurement of an « en-face » confocal image, i.e whole diagonal of the reflection matrix $(r_{in}=r_{out})$

Matrix approach of Full-field OCT

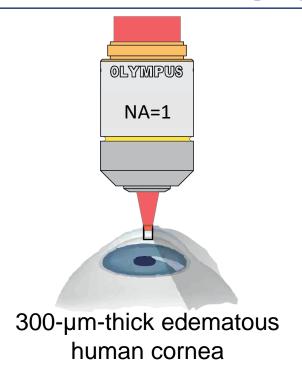




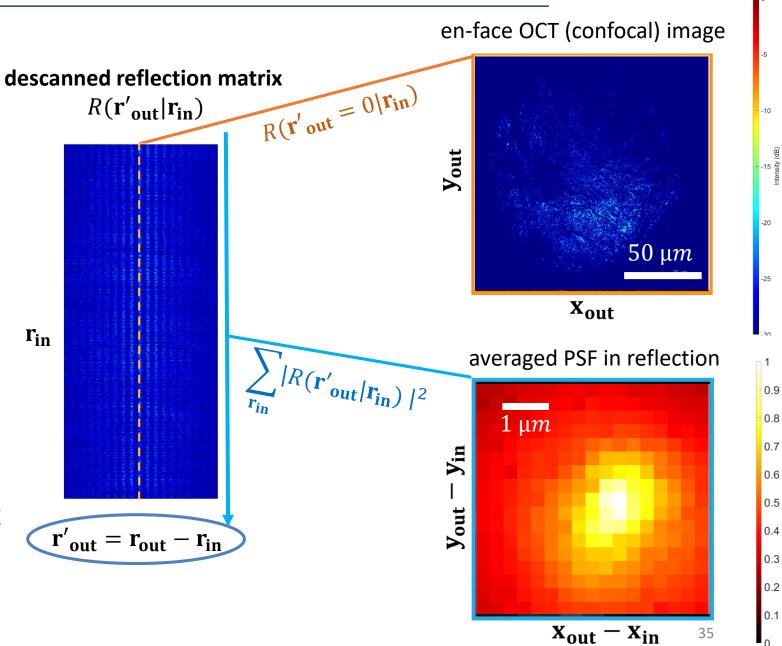
Parallel measurement of the R-matrix sub-diagonals

$$(r_{in}-r_{out}=\Delta r)$$

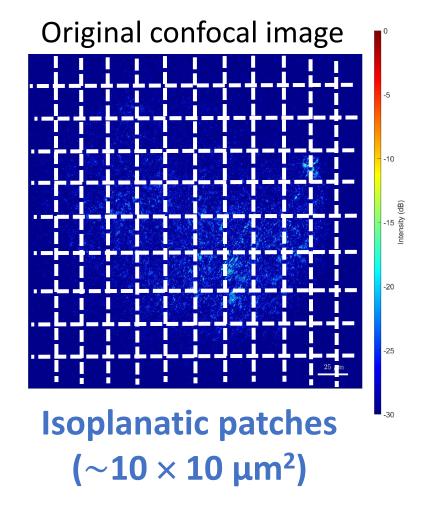
Passive matrix imaging



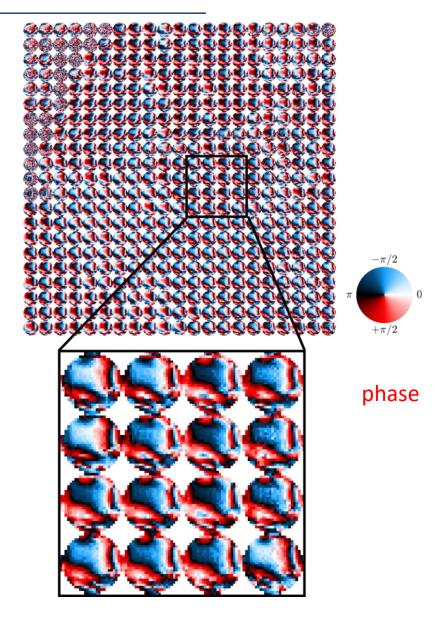
Field-of-view: $200 \times 200 \ \mu m^2$



Local distortion matrix analysis

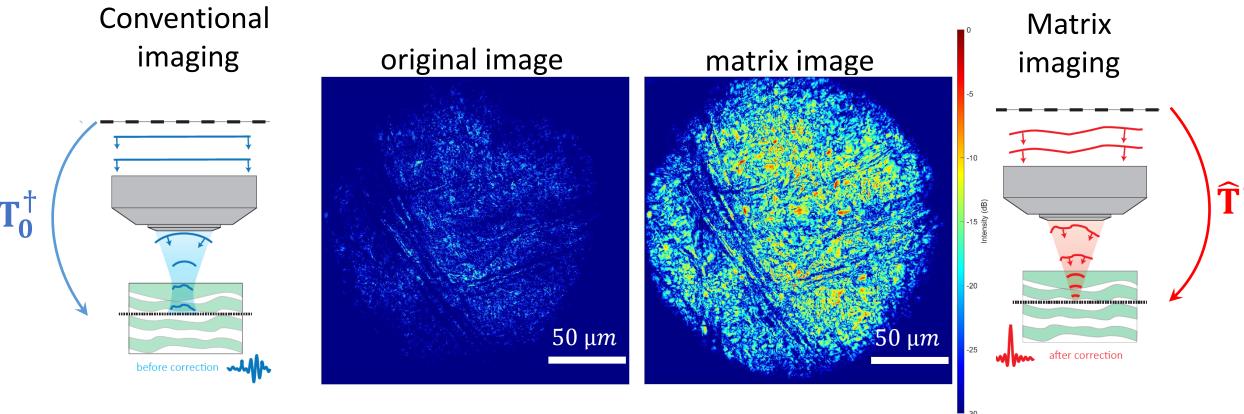






The isoplanatic assumption is everything but true in this configuration

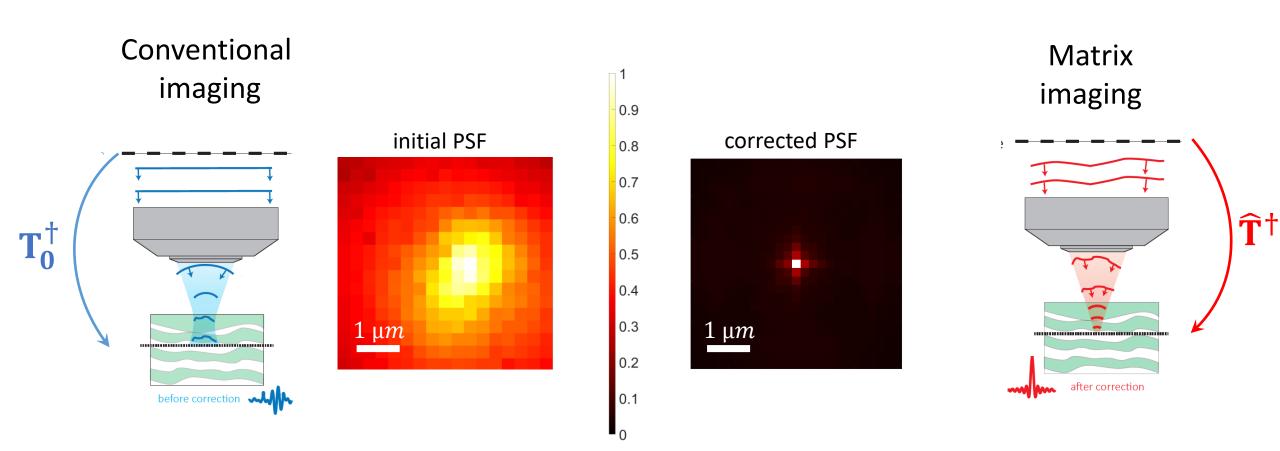
Transmission matrix correction



Drastic improvement of contrast: ×50 confocal intensity

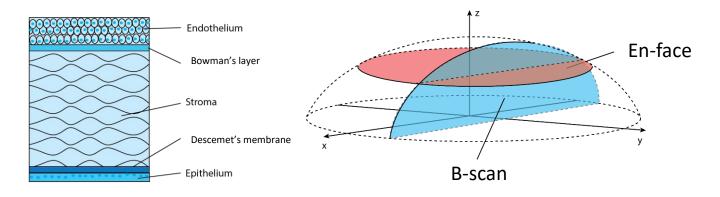
Stromal striae (indicator of keratoconus) are revealed by matrix imaging

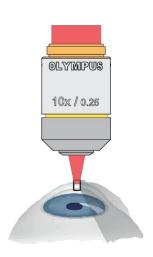
Transmission matrix correction

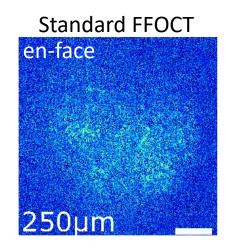


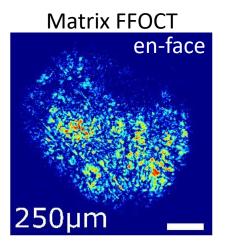
Drastic improvement of resolution (×10): 2.5 μ m \rightarrow 0.25 μ m

Passive matrix imaging

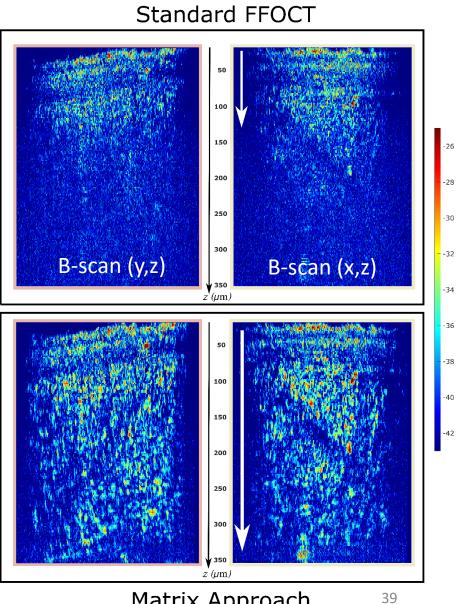








SNR × 30 in depth Penetration depth ×4



Matrix Approach



Passive matrix imaging:

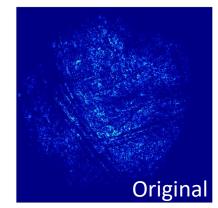
Non-invasive, label-free Huge FOV (10⁹ voxels) Confocal resolution (0.2 μm)

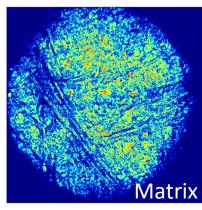
Penetration depth $\sim 10 I_s$

Specular / Speckle regimes

Overcome the intrinsic limit of adaptive optics (isoplanicity)







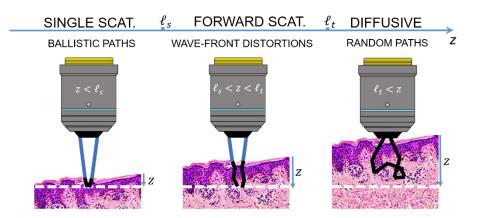
Limits:

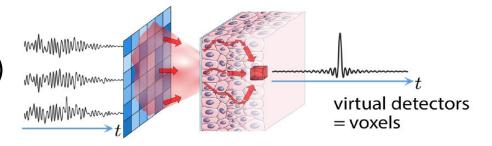
No correction of axial/dispersive aberrations Limited to single scattering and forward multiple scattering (< 1 I_t) Long acquisition time, low SNR

Perspectives:

Spectral domain OCT

Temporal/Frequency degrees of freedom (inspired by **ultrasound**) Extension to other fields of wave physics (**seismology**, radar *etc.*)





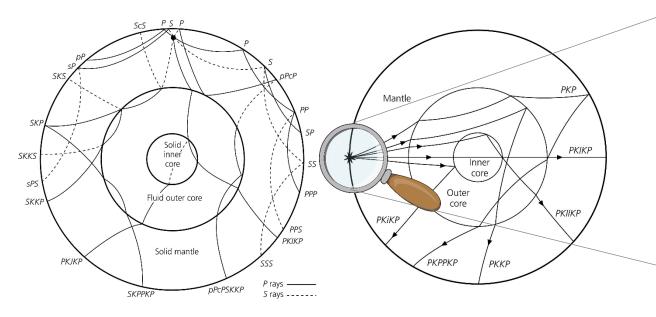
Passive matrix imaging

Application to seismology

Introduction: Reflection seismology



Seismic imaging at the *global* scale (« Transmission » configuration)

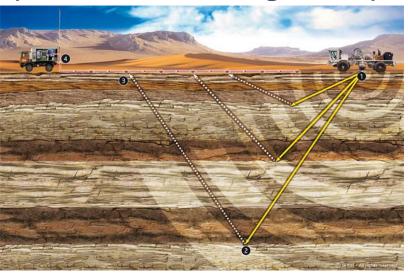


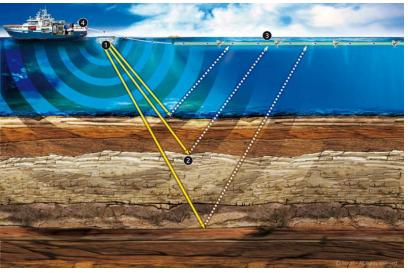
Stein, Wysession, *An Introduction to Seismology, Earthquakes and Earth Structure,*Blackwell Publishing 2003

The underground can be probed at the kilometer scale using man-made impulses

The underground heterogeneities **reflect** and **scatter** the seismic bulk waves as they propagate through

Seismic imaging at the *local* scale (« Reflection » configuration)

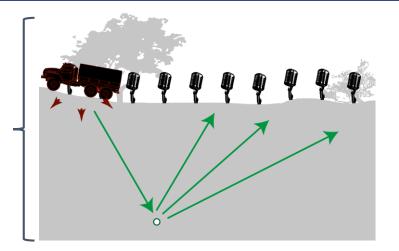




Introduction: Fundamental limits of reflection seismology



Homogeneous Earth's crust



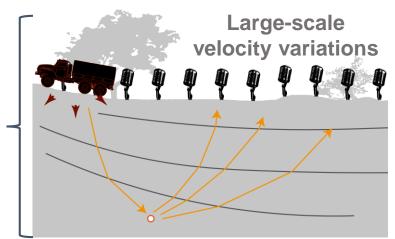
An incident wave is generated at the medium surface

The reflected wave-field is recorded by an array of geophones

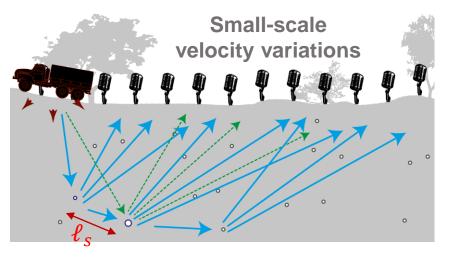
Single scattering regime:

Travel times directly yield the scatterer position

Heterogeneous Earth's crust



Aberrations: loss in contrast and resolution



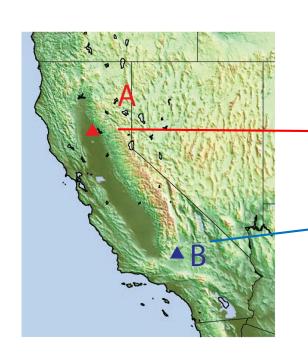
Multiple scattering: Penetration depth limited to ℓ_s

How to go beyond 1 km depth (no coherent source)?

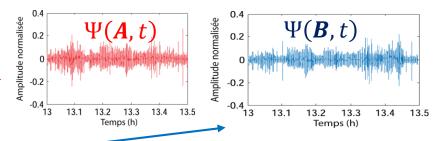
How to overcome multiple scattering and aberration issues?

Introduction: Green's function retrieval / Passive imaging





(1) Displacement recorded at two points A et B



Charcateristic time $\tau_c \sim 1$ s

(2) Green's function retrieval from noise cross-correlation

$$C(\mathbf{A}, \mathbf{B}, t) = \int_{0}^{T} d\tau \cdot \Psi(\mathbf{A}, t + \tau) \cdot \Psi(\mathbf{B}, \tau)$$

 $T \gg \tau_c : \partial_t C(A, B, t) = G(A, B, t) - G(A, B, -t)$

time reversal analogy / fluctuation – dissipation theorem

C

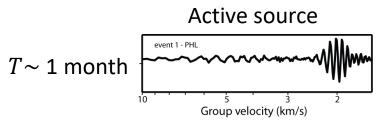
Causal Anti-causal

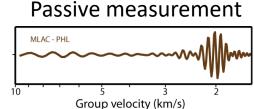
A. Derode et al., Appl. Phys. Lett. 83, 3054 (2003)

M. Campillo and A. Paul, Science 299, 5606 (2003)

N. M. Shapiro et al., Science **307**, 5715 (2005)

P. Roux et al., Geophys. J. Int. 206, 980-992 (2016)

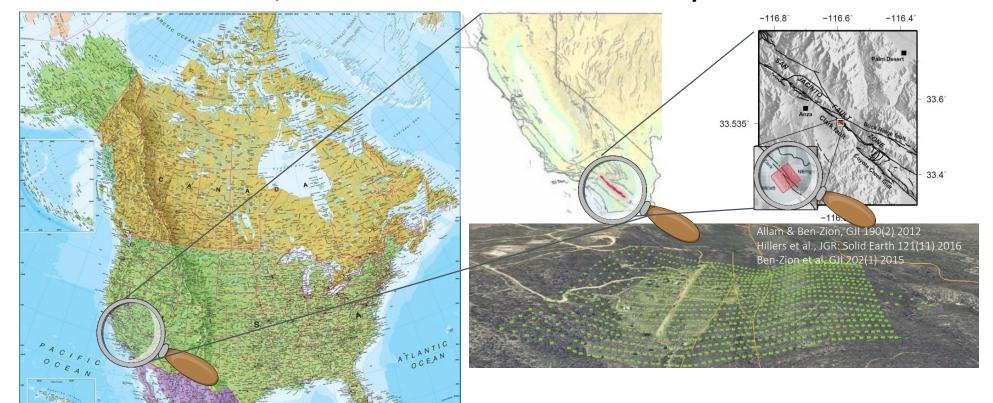




Passive imaging of the Earth's crust (mainly limited to Rayleigh surface wave tomography)

Passive matrix seismic imaging

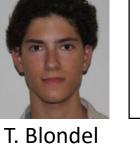
San Jacinto Fault Zone, one of the most seismically active areas in California





R. Touma





A. Derode





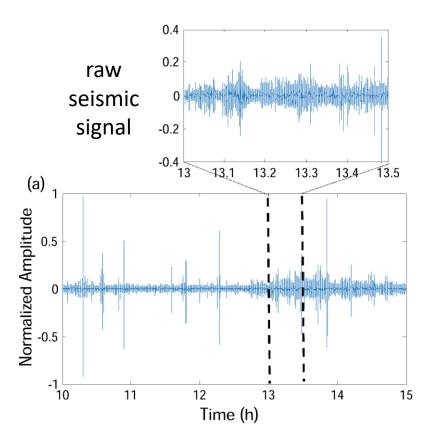
M. Campillo

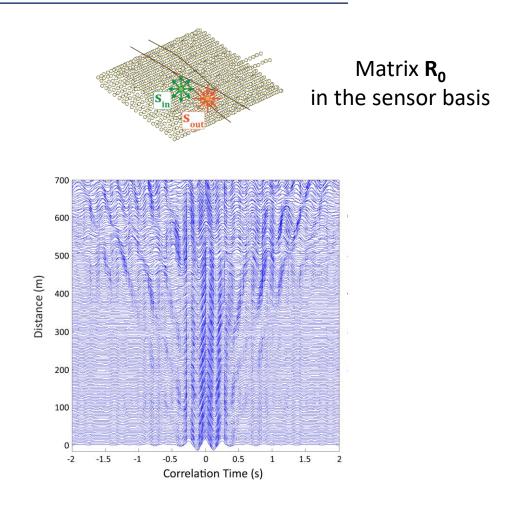
1108-geophones dense array ($\delta_X \approx 10 \text{m}$, $\delta_Y \approx 30 \text{m}$) on the San Jacinto Fault, California

 $f \sim 10 - 20 \text{ Hz}, \lambda \sim 50 - 200 \text{ m}$

Passive matrix seismic imaging

ocean, wind and human-related noise



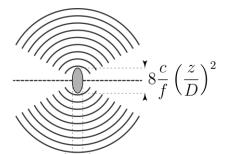


Passive measurement of the reflection matrix ${f R_0}$ between geophones by cross-correlation of seismic noise

Broadband focused reflection matrix

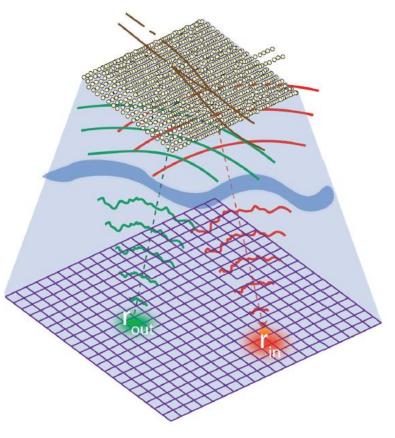
Monochromatic focusing

$$\mathbf{R}(z,\omega) = \mathbf{T}_{\mathbf{0}}^{\dagger}(\omega,z)\mathbf{R}_{\mathbf{0}}(\omega)\mathbf{T}_{\mathbf{0}}^{*}(\omega,z)$$



Free-space transmission matrix $T_0(\omega,z)$

Very rough velocity balground model: $c_0 = 1500 \text{ m/s}$



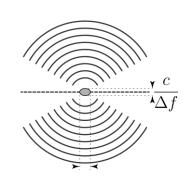
Broadband focused reflection matrix

$$\overline{\mathbf{R}}(z) = \sum_{\omega} \mathbf{R}(z, \omega)$$

Coherent sum over the frequency bandwith



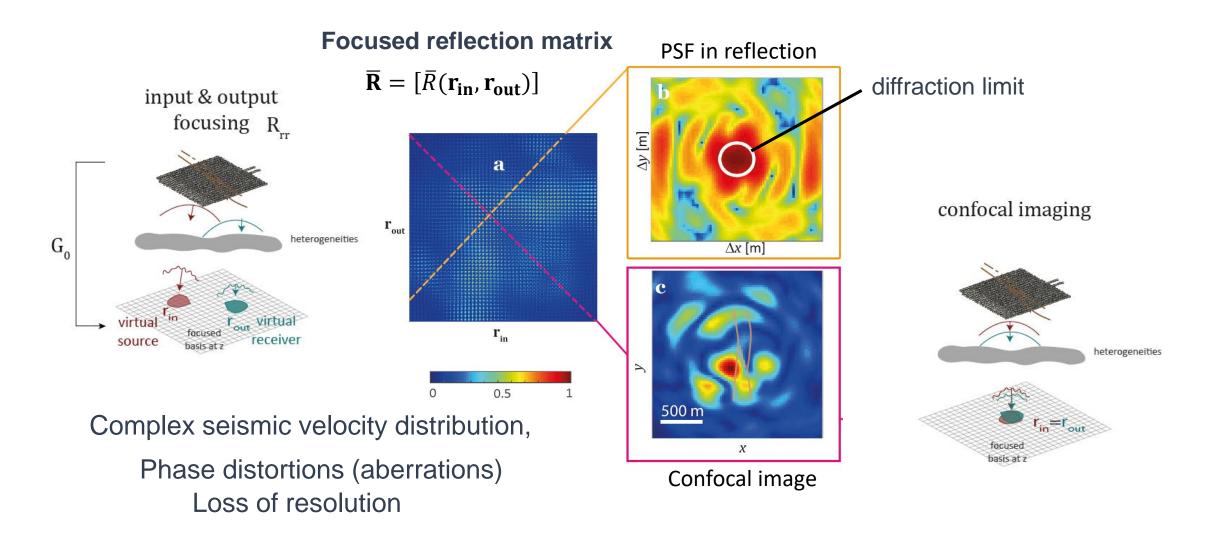
Numerical time gating



Projection of the reflection matrix in the focused basis (redatuming in seismology)

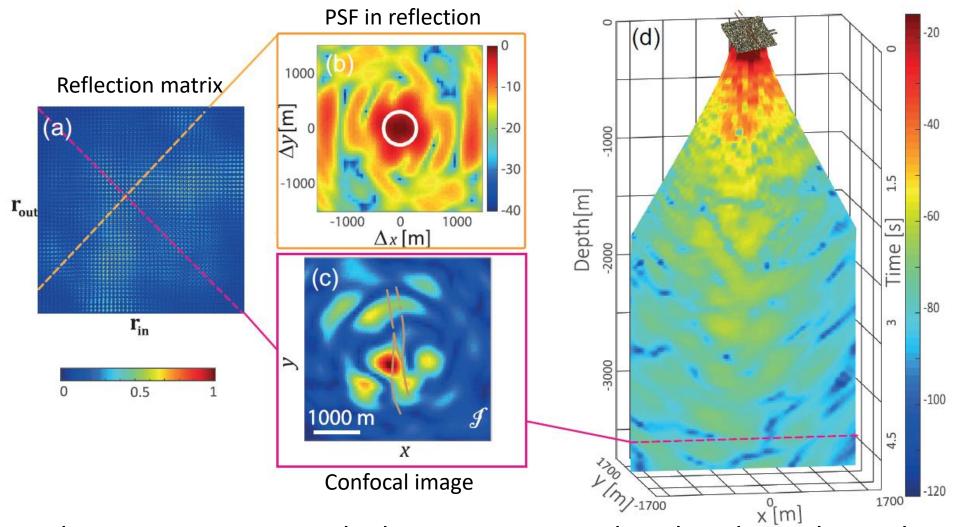
Berryhill, Geophysics, 1984
Berckhout and Wapenaar, J. Acoust. Soc. Am., 1993

Focused reflection matrix: Local aberration quantification



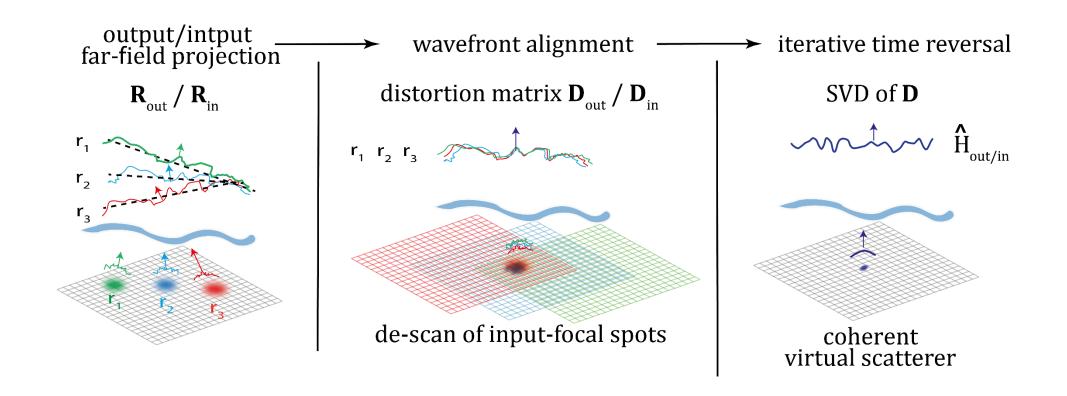
The focused reflection matrix enables a local assessment of the aberration level

Focused reflection matrix: Confocal imaging



The 2D images are stacked as 3D images and explored as volume slices However, the confocal 3D image cannot be trusted due to the high aberration level

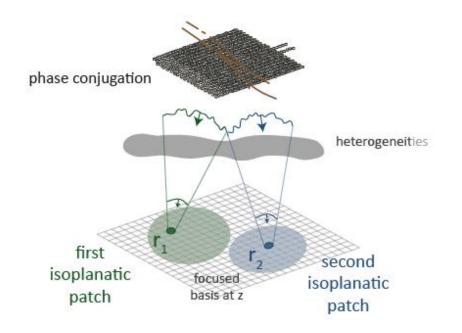
Distortion matrix approach for aberration correction

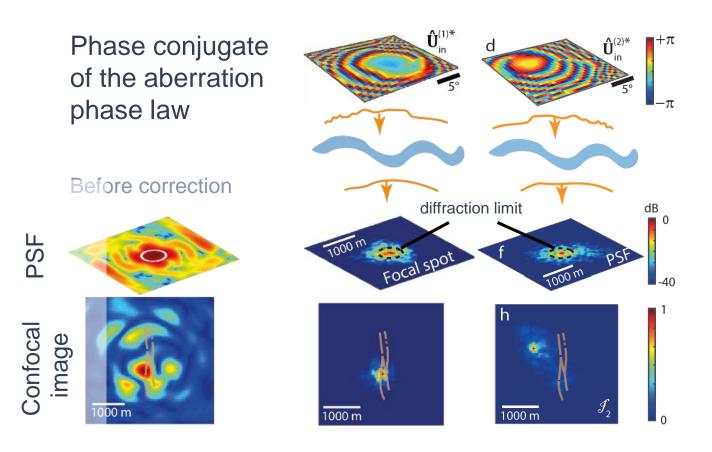


The distortion matrix yields an estimation of the aberration phase transmittance over each isoplanatic patch

Distortion matrix approach of seismic imaging

SVD of **D**:
One-to-one association between isoplanatic patches and aberration phase laws

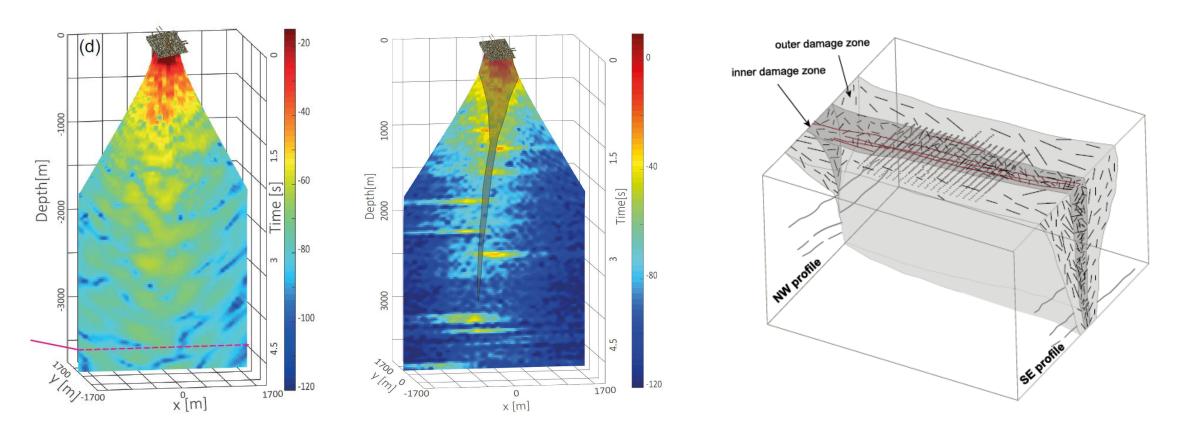




The aberration correction steers the focused energy back to the diagonal

Resolution almost 8× better than the diffraction limit in depth (free space)

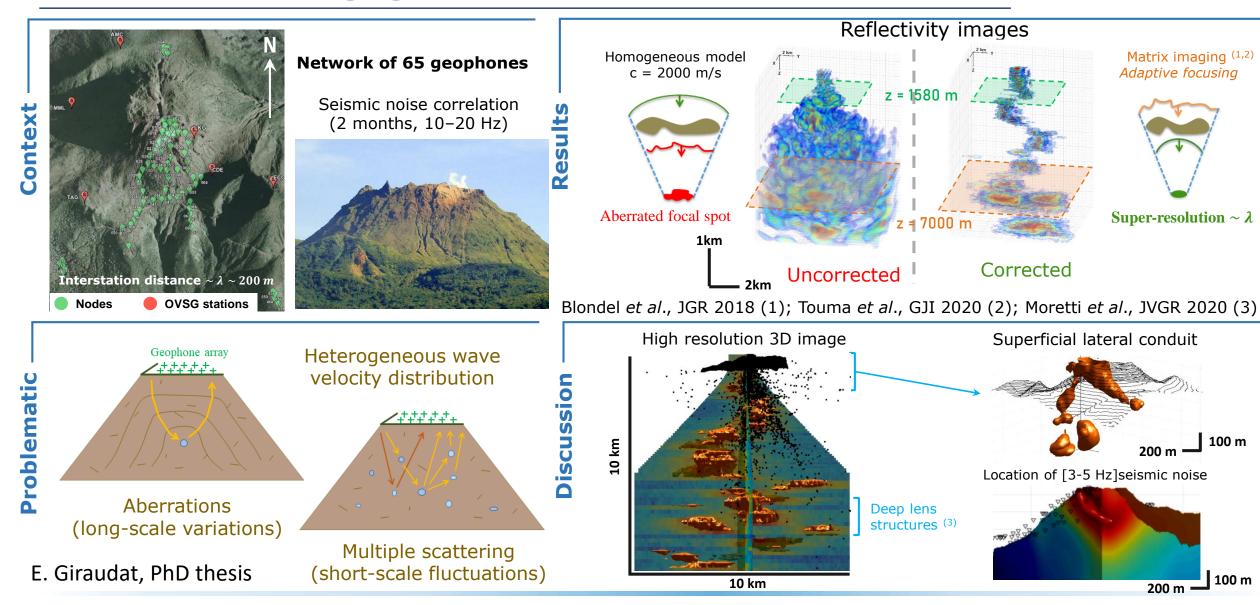
Input aberration correction



The matrix approach reveals the in-depth structure of the fault (shaded area):

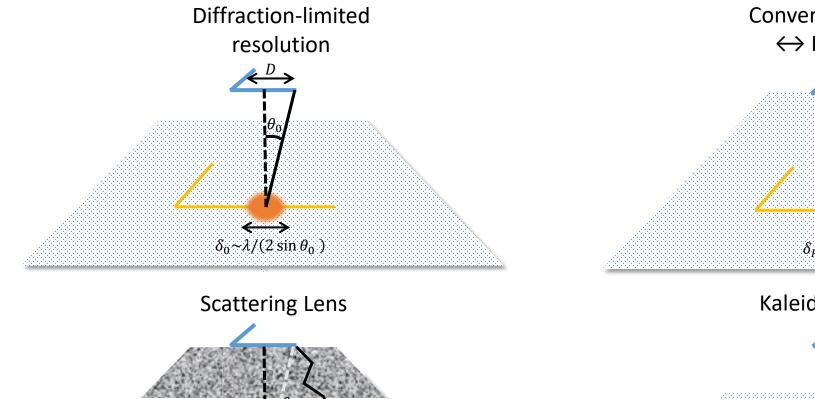
- -> Strongly heterogeneous damage area (z<1500 m)
- -> Offset between geological units at the fault location

Passive matrix imaging of La Souffrière volcano



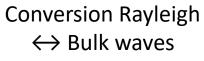
High resolution image of the volcano's internal structure by passive matrix imaging

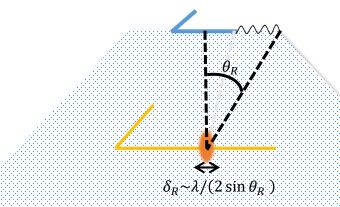
One open question: Physical origin for super-resolution?



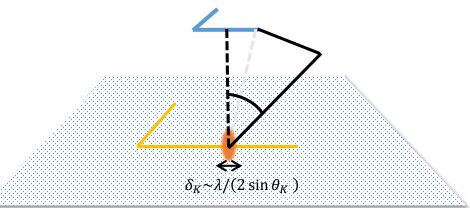
A. Derode and M. Fink, PRL, 1995

 $\delta_s \sim \lambda/(2\sin\theta_S)$





Kaleidoscopic Lens



P. Roux and M. Fink, J. Acoust. Soc. Am., 2001

Rayleigh waves, scattering and waveguiding are good candidates to account for super-resolution

Conclusion and perspectives



Passive matrix imaging:

Seismic noise, bulk waves
Fault zones, volcanoes
Dense network of geophones (Nyquist criterion)
Specular reflectors / Diffuse scattering
Penetration depth:
From 10 km (10-20 Hz) to 100 km (0.1-0.5 Hz)
Super-resolution ($\sim \lambda$)

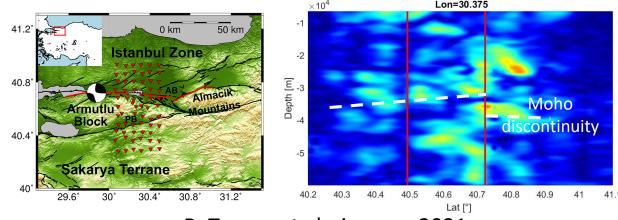
Limits:

No correction of axial/dispersive aberrations Conversion between P- and S-waves

Perspectives:

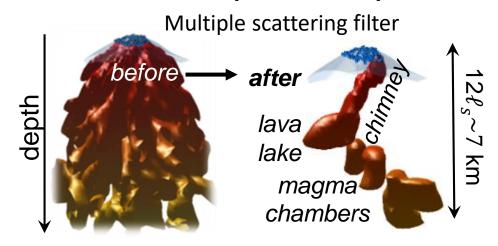
Bulk wave velocity tomography Exploit temporal degrees of freedom (inspired by ultrasound)

North Anatolian fault (Turkey)



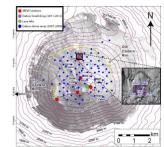
R. Touma et al., in prep. 2021

Erebus volcano (Antarctica)



T. Blondel et al., J. Geophys. Res. 2018

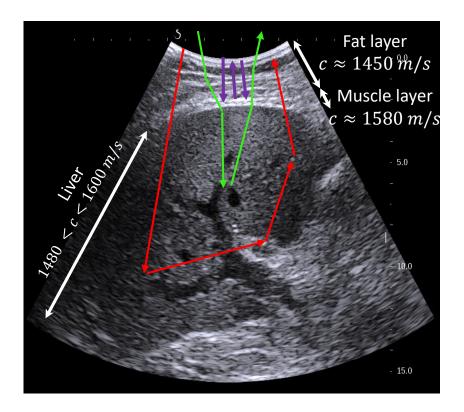




Towards Quantitative Matrix Imaging An ultrasound proof-of-concept

Matrix approach of ultrasound imaging

Ultrasound image (liver)



Conventional imaging

Homogeneous speed of sound « c »
Single scattering process
Scatterer position ⇔ echo time of flight
Confocal imaging

Aberration

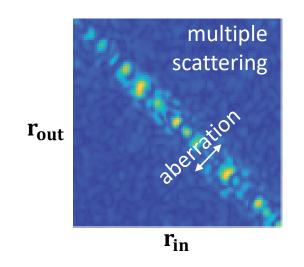
Resolution and contrast

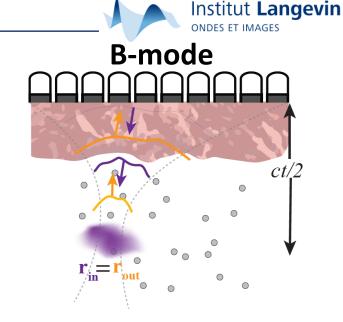
Multiple reflection

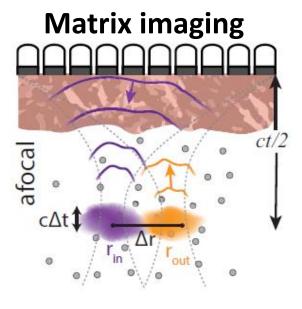
Reverberation artefacts

Multiple scattering

Incoherent background



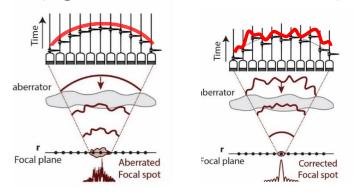


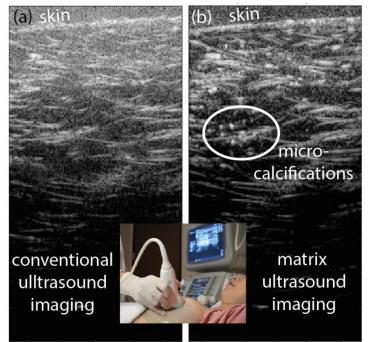


Ultrasound Matrix Imaging



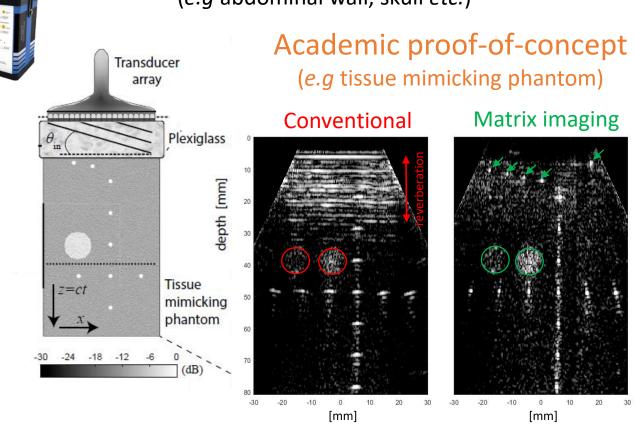
Imaging deeper and sharper (e.g breast micro-calcifications)





W. Lambert, PNAS, 2020

Remove/Compensate for Reverberations (e.g abdominal wall, skull etc.)



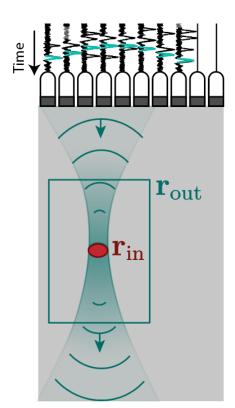
Contrast, transverse and axial resolution drastically improved by matrix imaging

E. Giraudat, PhD thesis

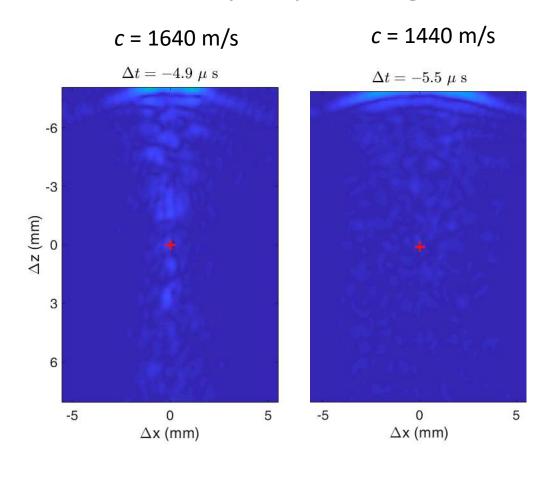


Self-portrait of an ultrasonic wave by exploiting scattering in tissues





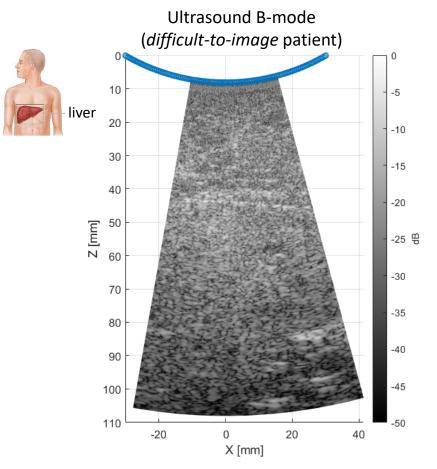
$$\overline{\mathbf{R}}(z) = \sum_{\omega} \mathbf{R}(z, \omega) \ e^{j\omega \Delta t}$$



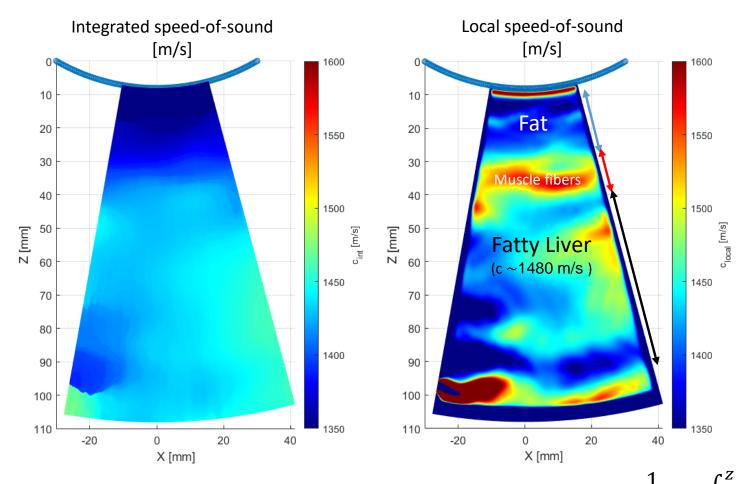
Measure the integrated speed-of-sound at a given pixel by tuning the axial position of the focusing point

Quantitative Matrix Imaging





Steatosis diagnosis:
Accumulation of fat droplets
within the liver cells $c_{\rm fat} \sim 1480 \; {\rm m/s} \; {\rm vs} \; c_{\rm liver} \sim 1600 \; {\rm m/s}$

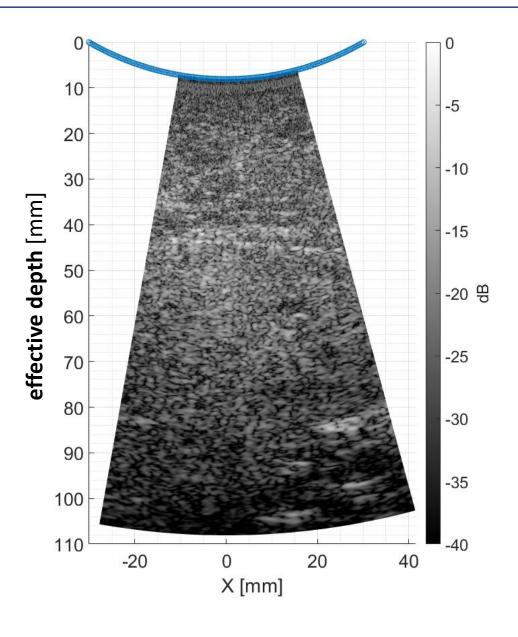


Tomography of the speed-of-sound,

an important parameter bio-marker for steatosis diagnosis

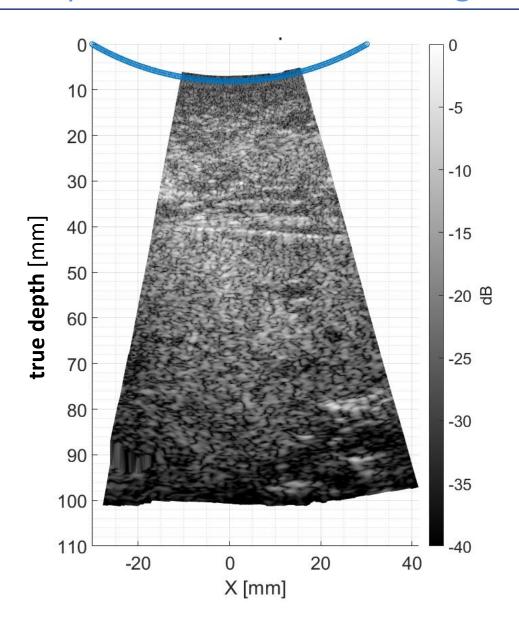
Exact repositioning in depth of the ultrasound image



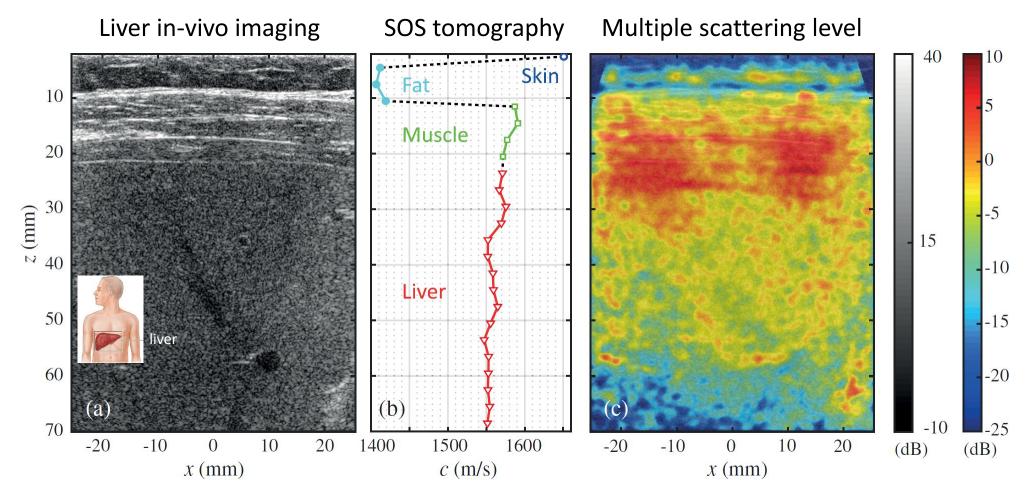


Exact repositioning in depth of the ultrasound image





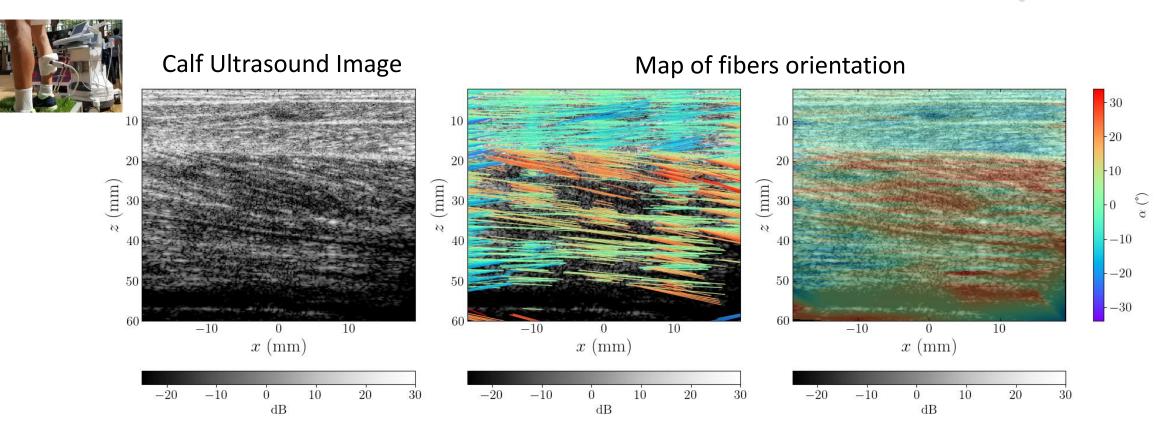




Tomography of multiple scattering, a new bio-marker for tumor detection?

Quantitative Matrix Imaging





Tomography of scattering anisotropy in fibrous tissues, an important bio-marker for muscular distrophy, myocardial disease *etc.*

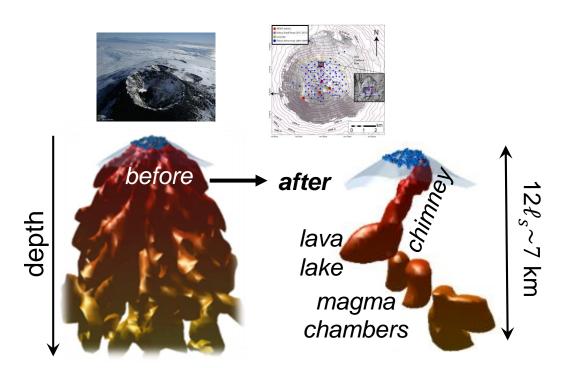


Conclusion & Perspectives

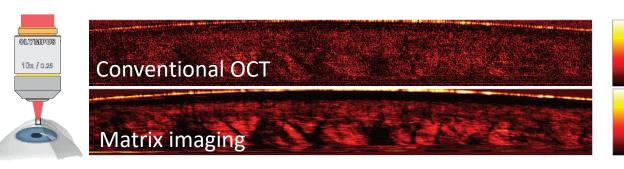
Universal matrix approach of imaging in wave physics

Reflection, Non invasive, Label free, Post-processing

Active/Passive measurements
Large dimension sensor networks
High number of spatial degrees of freedom



3D optical imaging of an edematous primate cornea



Applications to any research field:

- Optical microscopy
- Ultrasound imaging
- Seismology
- Radar technology
- etc.

But limited, so far, to

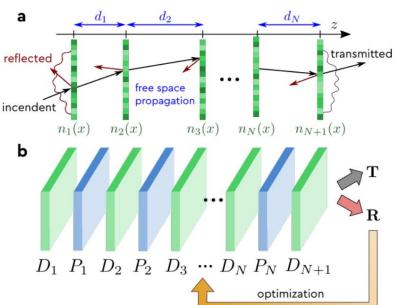
- single scattering
- forward multiple scattering...

Extend the matrix approach to the time/frequency domain

Go beyond the ballistic time

Control all the spatio-temporal degrees of freedom provided by the scattering medium

Self-portrait of the wave inside the scattering medium



Wavefront shaping Hoarstemeyer et al., Nat. Photon., 2015

Depth-by-depth iterative procedure

Deterministic and learning-based approaches for the retrieval of the T-matrix inside the medium

Courtesy of Sebastien Popoff

Collaborators

























William Lambert Laura Cobus

Flavien Bureau

Elsa Giraudat

Arthur Le Ber

Antton Goicoechea



Rita Touma







Arnaud Derode





Arnaud Burtin Michel Campillo



Thanks for your attention!

Institut Langevin, ESPCI Paris, CNRS, PSL University, Paris, France











